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TWO-STREAM PARAMETERIZATION OF THE FLUX DIVERGENCE IN A PLANE-P--ETC(U)

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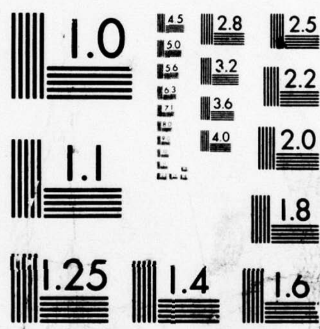

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Two-Stream Parameterization of the Flux Divergence in a Plane-Parallel Atmosphere

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Accurate two-stream representation of the angular distribution of radiation exchange in a plane-parallel atmosphere requires the use of a diffusivity factor at least 13% larger than the normally chosen values of 1.66 or $\sqrt{3}$ ($\cong$1.732), since the flux divergence, rather than the transmitted flux, determines the heating or cooling rates. Diffusivity factors are calculated to yield exact results at various optical depths for Doppler and Lorentz line profiles as well as for the grey-absorption case.

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TWO-STREAM PARAMETERIZATION OF THE FLUX DIVERGENCE IN A PLANE-PARALLEL ATMOSPHERE

1. INTRODUCTION

To speed up the solution of time-consuming radiative transfer problems in plane-parallel media the technique of the two-stream approximation is often adopted, particularly for planetary atmospheres.[1,2,3] The reciprocal of the cosine of the angle that represents the mean obliquity of the inward and outward streams is known as the "diffusivity factor," d_f. The two most common choices for its value are 1.66[1,2] and $\sqrt{3}$ ($\cong$ 1.732).[4] The value of 1.66[5] follows from the fact that a monochromatic isotropic radiation field is attenuated by one-half at a monochromatic normal optical depth of 0.42, that is, $0.5 = \exp(-1.66 \times 0.42)$. The $n = 2$ Gaussian quadrature integral approximation[4] yields a value of $\sqrt{3}$, which differs from 1.66 by only 4.3 per cent.

Two important atmospheric situations render these values inaccurate. First, in many cases of physical interest, the radiation field is highly anisotropic. Second, *net exchange* of radiative energy (as opposed to simple transfer of radiation from one layer to another) can occur between layers separated by optical depths much smaller or much larger than 0.42. For example, radiation exchange in the earth's atmosphere by the 15μ CO_2 bands generally occurs between layers separated by large optical depths since the atmosphere is isothermal on a scale of only 0.42 optical depths (less than 30 m at an altitude of 35 km). It should be emphasized that the flux divergence, rather than the transmitted flux, determines the net exchange.

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2. BASIS FOR CHOOSING THE DIFFUSIVITY FACTOR

Consider two differentially thin atmospheric layers at z' and z''. Monochromatic radiation emitted from the layer at z' toward the layer at z'' has an angle-averaged probability $\bar{P}_e$ of reaching z'' given by

$$\bar{P}_e = \frac{\int_0^1 j_z'(\mu) \exp\left(-\frac{\tau_{z',z''}}{\mu}\right) d\mu}{\int_0^1 j_z'(\mu)\, d\mu} \tag{1}$$

In equation (1), μ is the cosine of the angle of obliquity, $j_z'(\mu)$ is the emission coefficient at z', and $\tau_{z',z''}$ is the monochromatic normal optical depth between z' and z''. If j_z' is isotropic, i.e., *not* a function of μ, we find

$$\bar{P}_e = \int_0^1 \exp\left(-\frac{\tau_{z',z''}}{\mu}\right) d\mu = E_2(\tau_{z',z''}) \tag{2}$$

where E_2 is the familiar second exponential integral.

Clearly, the amount of radiation that originates at z' and is absorbed in a layer dz at z'' is proportional to $d\bar{P}_e/dz$ at z''. Let $P_e(\tau)$ be the *monodirectional* escape probability in the perpendicular direction ($\mu = \pm 1$). If a diffusivity factor $d_f(\tau)$ can be found which correctly represents $\overline{P_e(\tau)}$ by $P_e(d_f\tau)$, the angular dependence of the radiative transfer will be accurately parameterized by d_f. The fundamental importance of $\overline{P_e(\tau)}$ has been pointed out by Dickinson.[6]

The quantity $\bar{P}_e(\tau)$ is also equal to the factor by which the flux F is diminished across the optical path τ, where

$$F = 2\pi \int_{-1}^{+1} I(\mu)\mu\, d\mu$$

and $I(\mu)$ is the specific intensity. The angular distribution of I, i.e. its μ dependence, significantly affects the value of $\bar{P}_e$ across a given path. If $I(\mu) = I_0$ (isotropic) across a hemisphere, then, across path τ_0,

$$\begin{aligned} \overline{P_e(\tau_0)} &= F(\tau_0)/F(\tau = 0) \\ &= \int_0^1 I_0 \exp\left[-\frac{\tau_0}{\mu}\right] \mu d\mu/(I_0/2) \\ &= 2E_3(\tau_0) \end{aligned} \tag{3}$$

Such an isotropic flux would be produced by a Lambert surface. However, the flux emitted from an optically thin layer of optical depth Δ in the atmosphere is characterized by a highly *anisotropic* $I(\mu)$. In fact, $I(\mu) = S\Delta(\mu) = S\Delta/\mu$ where S is the source function j/k. The corresponding flux diminution $\overline{P_e(\tau_0)}$ is equal to

$$\begin{aligned} F(\tau_0)/F(0) &= \int_0^1 (S\Delta/\mu)\, e^{-\tau/\mu}\, \mu d\mu \Big/ \int_0^1 (S\Delta/\mu)\mu d\mu \\ &= E_2(\tau_0), \end{aligned} \tag{4}$$

precisely the result of equation (2). The physical content of equations (2)-(4) may be summarized as follows. The escape probability (flux transmission) across a layer equals $2E_3$ if the radiation incident upon the layer is isotropic. Heating or cooling due to exchange between layers is determined by the transmission of radiation among differentially thin layers whose escape probability across a finite layer equals E_2. The fact that the integrated radiation field from an optically thick finite portion of the atmosphere may be nearly isotropic is not relevant since it is the *derivative* of the $2E_3$ transmission function ($\propto E_2$) for isotropic radiation which determines the exchange. Thus diffusivity factors based upon E_2 rather than $2E_3$ should be used whenever the calculation of exchange phenomena such as heating or cooling is the objective.

If radiation is transferred primarily within lines the most useful quantity to consider is the line-profile averaged, angle-averaged escape probability

$$\bar{P}_{e\ d\ or\ l} = \int_{-\infty}^{+\infty} \phi(x)\ \bar{P}_e(\tau_x)\ dx \tag{5}$$

In equation (3) x is the frequency variable, $\phi(x)$ is the line emission profile, assumed proportional to the absorption profile, and normalized such that

$$\int_{-\infty}^{+\infty} \phi(x)\ dx = 1 \tag{6}$$

The subscripts d or l correspond to the $\bar{P}_e$ for a Doppler or Lorentz line profile, respectively. For a Doppler profile $\phi(x) = \frac{e^{-x^2}}{\sqrt{\pi}}$; for a Lorentz profile $\phi(x) = 1/\pi(1 + x^2)$.

3. NUMERICAL RESULTS

The integrals of Eqs. (2) (for monochromatic radiation) and (5) (for both Doppler and Lorentz profiles) were computed numerically and compared with the monodirectional escape probabilities to find the diffusivity factor required for *exact* agreement with $\bar{P}_e$. In Fig. 1 the line profile-averaged escape probabilities are plotted for Doppler and Lorentz lines for optical depths from 10^{-1} to 10^3. Figure 2 displays the required diffusivity factors up to 7 optical depths at line center, the most interesting region of variation. Also shown in Fig. 2 is the diffusivity factor for monochromatic radiation (frequency-independent absorption). For $7 \leqslant \tau \leqslant 10^3$, $d_f = 1.95$ for Doppler lines, while the d_f increases from 2.22 at $\tau = 7$ to 2.24 at $\tau = 10^3$ for Lorentz lines. For grey absorption (monochromatic radiation) d_f slowly approaches 1 as $\tau \to \infty$.

The limits of d_f as $\tau \to \infty$ may be straightforwardly derived from equation (5). Integrals similar to equations (2.21) and (2.27) of Holstein[7] are obtained and easily solved by his technique as $\tau \to \infty$. The Doppler diffusivity factor slowly approaches 2 as $\tau \to \infty$ while for Lorentz lines the limit is $d_f = \left(\frac{3}{2}\right)^2 = 2.25$. For $\tau = 0$, a diffusivity factor based on flux

transmission is 2.0[8], whereas a diffusivity factor appropriate for flux divergence approaches $+\infty$. The latter indicates that only that portion of the radiation field which traverses a long optical path (i.e., large oblique angles) can cause heating or cooling as $\tau \rightarrow 0$.

4. SUMMARY AND CONCLUSIONS

If the primary objective of a radiative transfer calculation is to obtain radiative exchange rates rather than flux transmission, a diffusivity factor must be chosen to match the flux divergence rather than the flux. It has been shown that these factors for an optically thick atmosphere should range from 1.95 for a pure Doppler profile to 2.25 for a pure Lorentz profile. These values are significantly higher than the usual value of 1.66 for heating calculations.[2] The need for larger diffusivity factors has been confirmed by the multiple scattering continuum heating calculations of Lacis and Hansen,[3] who found that $d_f = 1.9$ provides the best fit to their detailed numerical work. This value is exactly the diffusivity factor required for optical depth unity and grey absorption (Fig. 2), where the bulk of continuum transfer effects occur.

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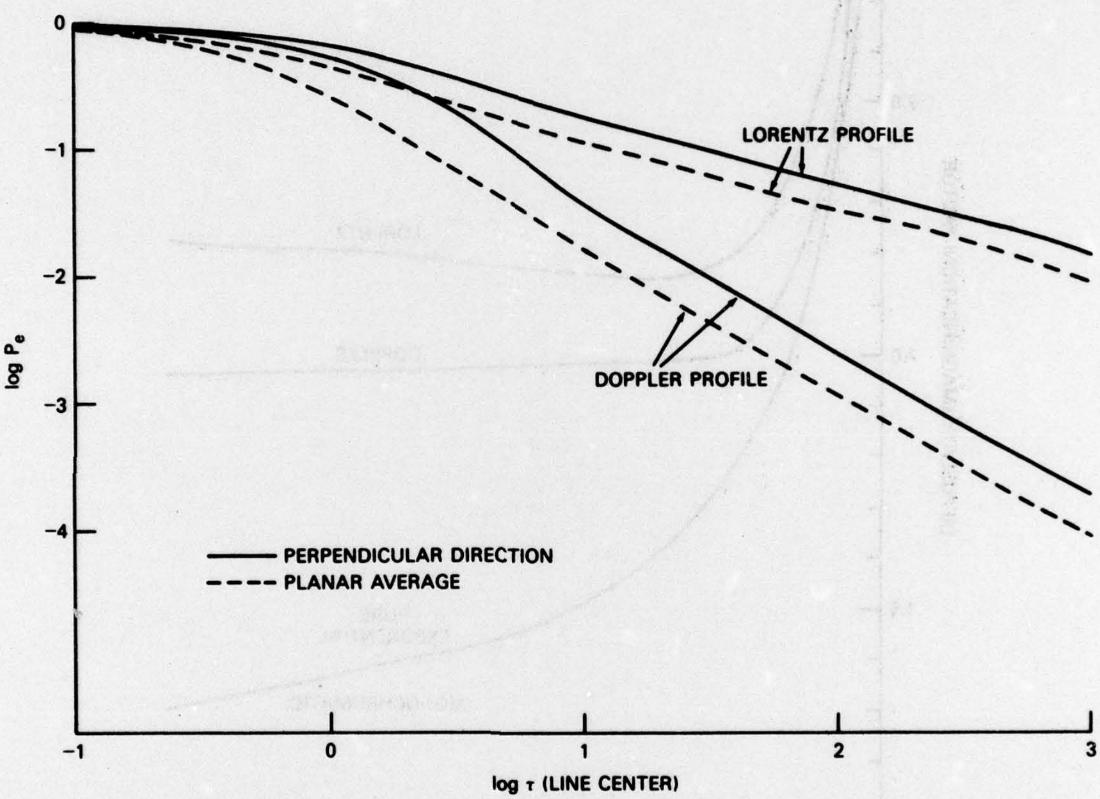

Figure 1 — Line-profile-averaged escape probabilities are logarithmically plotted against optical depth, for pure Doppler and Lorentz profiles. Values are given for the connecting path perpendicular to the planes, and for the angle-averaged escape probability.

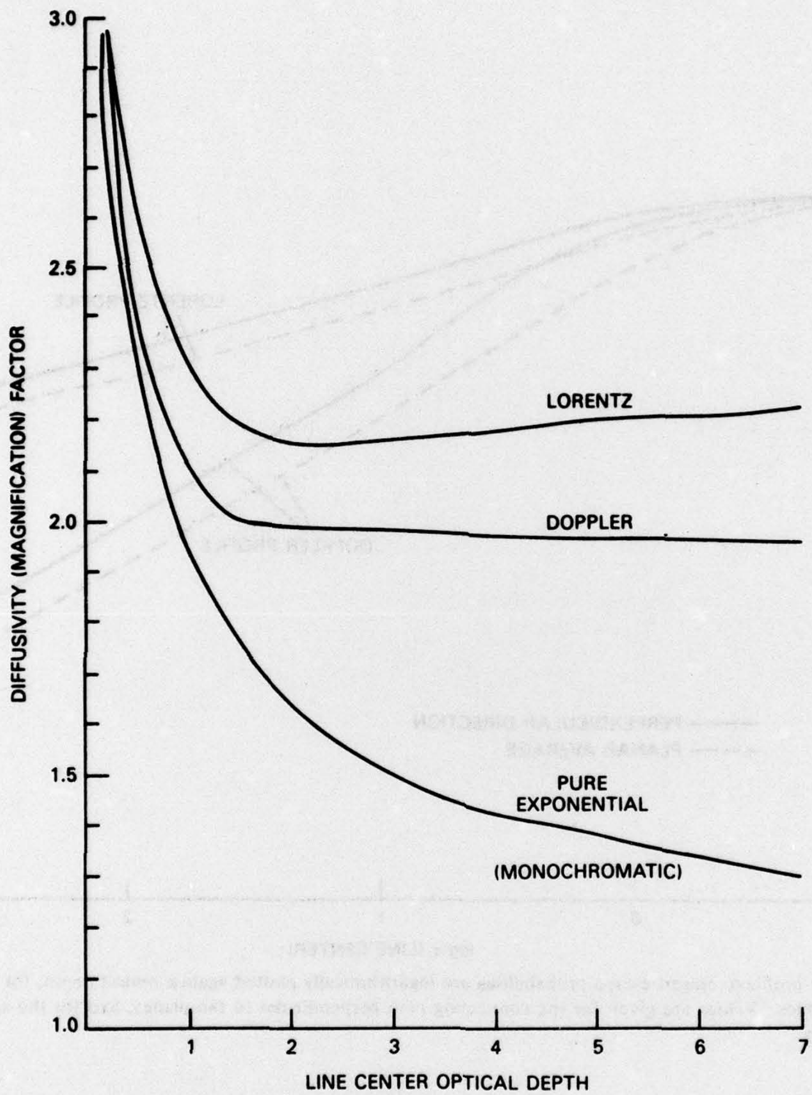

Figure 2 — The required diffusivity factor is given for line-profile-averaged escape probabilities for Doppler and Lorentz lines, and monochromatic radiation, as a function of optical depth.